PRIMER ON MONOMIAL IDEALS AND IDEAL DECOMPOSITION

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Abstract. This document contains some of the basic facts and observations about Monomial ideals. Know them! And know how to prove them.

Theorem 1. *All Prime ideals are primary. All prime ideals are irreducible. In a Noetherian ring all irreducible ideals are primary.*

Definition 1. A *primary decomposition* is a decomposition of I in the finite intersection of primary ideals, i.e $I = I_1 \cap \ldots \cap I_n$ where each I_k is primary

Theorem 2. *$\sqrt{I}$ is prime, so $\sqrt{I} = P$ where P is prime. I is thus called P-primary*

Theorem 3. *If Q P-primary and $f \in R$ then*

$$f \in Q \implies Q : f = R$$
$$f \notin Q \implies Q : f \text{ is } P\text{-primary}$$
$$f \notin P \implies Q : f = Q$$

Theorem 4. *The colon and radical distribute over intersection, i.e $(I \cap J) : f = (I : f) \cap (J : f)$ and $\sqrt{I \cap J} = \sqrt{I} \cap \sqrt{J}$*

Theorem 5. *The set of associated primes of I as an ideal is called $Ass(I)$. $\{\sqrt{I : f}\}_{f \in R} = Ass(I)$*

Theorem 6. *The set of associated primes of M as an R-module is called $Ass(M)$. $Ass(M) = \{I \subseteq R \,|\, I = ann(m)\, \forall m \in M\}$*

Theorem 7.

Definition 2. A *monomial ideal* is an ideal $I \subset K[x_1, \ldots, x_n]$ generated by monomials, i.e $I = \langle \mathbf{m}^{\alpha_1}, \ldots, \mathbf{m}^{\alpha_n} \rangle$

Criterion 8. *I monomial ideal $\iff \forall\, f \in I, supp(f) \in I$*

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Criterion 9. *I monomial ideal $\iff \forall\, f \in I, supp(f) \in I$*